

VEHICLE POOLING IN TRANSIT OPERATIONS

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ABSTRACT: The benefits of pooling vehicles among routes that emanate from a common focus terminal are examined. In this strategy, trips are still scheduled, but vehicles are not assigned to specific trips. Instead, vehicles belonging to the pool serve all of the round trips leaving that terminal in a first in/first out sequence. Pooling improves schedule adherence, since in a pooled system a bus returning early can "cover" for a bus returning late. Pooling also facilitates interlining (sharing of buses among routes), which reduces the need for slack time. A procedure is developed for estimating schedule reliability. This procedure is applied to a set of 8 routes emanating from a Boston area terminal where it was found that with pooling the fleet size could be reduced by 11% while at the same time improving schedule adherence.

DESCRIPTION OF VEHICLE POOLING STRATEGY

Customary practice in transit scheduling is for each trip to be scheduled in time and for each vehicle to be scheduled to serve a fixed sequence of trips. In many cases, during a given period such as the evening peak, vehicles serve on one route only, making each route an independent operation. In other cases, vehicles are interlined, i.e., serve on more than one route, but each vehicle has its own fixed schedule of trips nonetheless. This paper describes a different operations strategy called "vehicle pooling" that can be applied to serve a set of routes that share a common terminal. In this strategy, a number of vehicles are assigned to a pool focused at a particular terminal. The vehicles in the pool are collectively responsible to serve all the trips emanating from (and returning to) the focus terminal. The trips are all scheduled as in conventional operation, but vehicles are not assigned to particular trips. Rather, vehicles are under the control of a dispatcher at the focus terminal whose task is simply to select and dispatch one vehicle from the pool of available vehicles whenever the time for a scheduled departure arrives. Trips on which a vehicle is dispatched are round trips, bringing the vehicle back to the focus terminal. (If no vehicles are available for a scheduled departure, the dispatcher must exercise some sort of remedial control, just as in an unpooled system.) The vehicle driver's duty in a pooled system is simply to make the round trip on which he is dispatched and then, upon returning to the focus terminal, await another dispatch. Since in this strategy trips are still scheduled, the public should perceive no change in operations except that the face of the bus driver will change from day to day.

To a small extent, the pooling strategy is employed in American transit systems. If one route is crippled due to vehicle breakdowns or traffic

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delays, the dispatcher can override vehicle schedules by transferring some buses to the crippled route. Sometimes a small number of buses is permanently assigned to "stand by" at a particular terminal in order to cover for trips that would otherwise be missed; these stand-by vehicles constitute a small vehicle pool. Even with a small pool of stand-by vehicles, however, most of the vehicles have a fixed schedule, and it is natural that the dispatcher should try to keep adjustments to the vehicle schedule as small as possible. By having all of the vehicles that operate out of a given terminal serve essentially on stand-by, the vehicle pooling strategy gives the dispatcher maximum flexibility in assigning vehicles to trips without the restriction of a fixed vehicle schedules so that all the scheduled departures can be served more reliably.

BENEFITS OF VEHICLE POOLING

The main benefits of vehicle pooling arise from two factors. First, pooling can be expected to lead to a greater degree of interlining than occurs in customary vehicle scheduling. Second, pooling increases the reliability of the operation as the on-time departure of each scheduled trip is not dependent upon a single vehicle being available for it.

Interlining Benefit.—Because of the integer nature of vehicles, a considerable amount of slack will usually exist in a vehicle schedule if vehicles are not interlined. For example, without interlining, a route with an 84-min run time that operates at 20-min headways must incorporate a slack of 16, 36, or, in general, $16 + 20k$ min, in which k = a non-negative integer. While a certain amount of slack is necessary to allow for recovery time, the integer constraints do not generally make it possible for the slack to be kept at its minimal level. Thus, for example, a route whose trip schedule demands 4.8 vehicle hours of operation/hr and requires a minimum 10% slack for recovery time could be served with 5.3 vehicle hours/hr if no extra slack were included in the schedule, but would need 6 vehicles (an integer) without interlining. By interlining two routes that share a common terminal and whose schedules are compatible, the integer constraint can be made to apply to the pair and not to each route separately, reducing the amount of schedule slack. For example, if a second route operating out of the same terminal and at the same headway needed 3.4 vehicle hours/hr for running and recovery time, the two routes if not interlined would require $6 + 4 = 10$ vehicles, but if interlined would require only 9 vehicles. By interlining all the routes that share a common terminal, possibilities for efficiently meshing schedules increase, even if the routes operate at different headways. In addition, the integer constraint will apply to the whole group of routes, thus possibly reducing still further the amount of slack, leading to further vehicle savings.

Deriving efficiency through interlining is a goal of both manual and recent automated scheduling methods, such as those described in Refs. 1 and 2. However, taking the maximum possible advantage of interlining in conventional operations can result in very complex vehicle schedules. Because it is difficult both for schedulers to develop complex schedules and for dispatchers to grasp them well enough to make good day-to-day adjustments to them, the extent to which interlining is incorpo-

rated in customary vehicle schedules probably falls short of achieving the maximum possible reduction in schedule slack, even when scheduling is automated.

In contrast to conventional operations, under the pooling strategy all the routes served by the pool are effectively interlined, and complex patterns of interlining are no more difficult to create or operate than simple patterns. Therefore, we should expect a greater degree of interlining to occur in a pooled system, with a consequently greater reduction in unnecessary schedule slack.

Swapping Benefit.—By enabling vehicles to "cover" for each other, vehicle pooling increases reliability, defined in this context as the probability of all scheduled departures being made on time. This benefit is best demonstrated by an example. Suppose departures 1 and 2 are scheduled for times D_1 and D_2 at the same terminal. In an unpooled system, each departure would be scheduled as a part of a block of trips served by a single vehicle. The trip that precedes trip i in its block of trips is called the predecessor of trip i . If t_i denotes the time at which trip i 's predecessor is completed, then the reliability, R , (the probability that both departures will be made on time) is

$$R = P[(t_1 \leq D_1) \cap (t_2 \leq D_2)] \quad (1)$$

If instead vehicles 1 and 2 are pooled for serving departures 1 and 2, the reliability is

$$R = P[(t_1 \leq D_1 \cap t_2 \leq D_2) \cup (t_2 \leq D_1 \cap t_1 \leq D_2)] \quad (2)$$

Assuming that t_1 and t_2 are independent, the reliability in the unpooled case is $P(t_1 \leq D_1)P(t_2 \leq D_2)$, while the reliability in the pooled case can be shown to be

$$P(t_1 \leq D_1)P(t_2 \leq D_2) + P(t_2 \leq D_1)P(D_1 \leq t_1 \leq D_2) + \dots \quad (3)$$

The second term in Eq. 3 is the reliability gain derived from pooling. It reflects the event that vehicle 2 arrives in time to make the first departure and vehicle 1, while arriving too late for the first departure, arrives in time for the second.

To demonstrate these results numerically, suppose t_1 and t_2 are normally distributed with means $(D_1 - s)$ and $(D_2 - s)$, respectively, in which s represents schedule slack. Suppose both t_1 and t_2 have a standard deviation of $\sigma = 5/k$. Then if Φ = the cumulative standard normal distribution function, the unpooled reliability is $[\Phi(k)]^2$ and the pooled reliability is

$$R = [\Phi(k)]^2 + \Phi\left[\frac{k-k(D_2 - D_1)}{\sigma}\right] \left\{ \Phi\left[\frac{k+k(D_2 - D_1)}{\sigma}\right] - \Phi(k) \right\} \quad (4)$$

If, e.g., $k = 1$ and the offset between the two departures, $(D_2 - D_1)$, equals the schedule slack, then the unpooled reliability is 0.707 while the pooled reliability is 0.775. Pooling two vehicles to cover two departures in this case increases the probability of on time performance by about 0.07.

The improved reliability demonstrated by this simple example comes about because, relative to a system with fixed vehicle schedules, pooling

creates more ways in which a trip schedule can be met. While in the example only one additional way was created (vehicle 2 making the first departure and vehicle 1 making the second), pooling many vehicles to cover many trips will create many new ways to meet a given trip schedule, each of which adds a little to the overall system reliability.

ESTIMATING RELIABILITY OF POOLED SYSTEM

As pointed out earlier, each way of meeting the trip schedule contributes to the probability that the schedule will be kept. In even a moderately sized system, the number of ways is prohibitively large. Suppose each trip is given an index, i , and suppose that each trip i has a certain number of trips, u_i , that could be trip i 's predecessor, i.e., the trip performed prior to trip i by the bus that is dispatched on trip i . Then if there are m trips to be served during the time period of analysis, there are $\prod_{i=1}^m u_i$ ways of meeting the schedule. If $u_i = 4$ for all i and if $m = 20$, there will be 4^{20} ways to keep the schedule, and thus 4^{20} terms in the reliability sum. Therefore, it is necessary to estimate the reliability for all but the smallest systems.

One important simplifying assumption used in this research is that travel times on different trips are independent. Strong correlation of travel times of all the trips on a single route is to be expected when there is a major crisis on that route, such as a lane closing or an accident. However, in normal operations, the random delays caused by demand and traffic fluctuations show less dependence across trips both of different routes and of the same route.

The approach taken to estimating pooled reliability was to compute a lower bound by summing only a subset of terms belonging to the reliability sum. The included terms are the reliability contributions of certain categories of ways the trip schedule can be met. These categories are constructed in a hierarchy as follows.

1. Base schedule.-A base schedule, with each vehicle assigned to a particular sequence of trips as in an unpooled system, is constructed using 80 percentile run times with a first in/first out heuristic. The first term in the pooled probability estimate is then the probability of being able to keep this base schedule. This probability is the unpooled reliability of the base schedule, R_u , given by

$$R_u = P[(t_1 \leq D_1) \cap (t_2 \leq D_2) \cap \dots \cap (t_m \leq D_m)] \quad (5)$$

in which t_j = completion time of the trip that, in the base schedule, is meant to precede trip i , assuming that the predecessor trip began on time; D_i = scheduled departure time of trip i ; and m = number of trips.

Calling $r_j = P(t_j \leq D_j)$ the reliability of trip i , and assuming independence among trips, the unpooled reliability can be expressed as

$$R_u = \prod_{i=1}^m r_j \quad (6)$$

If, for the given number of buses, the probability of meeting the base schedule constructed using 80 percentile times is zero, a new base schedule

can be constructed using 60 percentile run times, or a still smaller percentile if necessary. In most cases, the unpooled reliability of the base schedule will be the largest term in the pooled reliability estimate, and thus, it is worth some computational effort to find a base schedule with a good unpooled reliability.

The remaining ways of keeping the trip schedule are all modifications of the base schedule. They are structured hierarchically, with each way incorporating the condition that earlier listed ways of keeping the trip schedule could not be met, thus ensuring that all the ways are mutually exclusive, so that their probabilities can be summed.

2. Nearest neighbor (NN) swap.-The base schedule cannot be kept, but all departures can be made on time if the bus that, in the base schedule, was assigned to trip k (bus k) serves instead trip $k + 1$. At the same time, the bus that in the bus schedule was assigned to trip $k + 1$ (bus $k + 1$) serves trip k . The probability of this event, P_{NNk} , is

$$P_{NNk} = \frac{P(t_{k+1} \leq S_{Dk}) P(D_k < t_k : S_{Dk+1}) R_u}{(r_k r_{k+1} + d)} \quad (7)$$

The total probability of all simple NN outcomes, R_{NN} , is

$$R_{NN} = \sum_{k=1}^{m-1} P_{NNk} \quad (8)$$

3. Pair of NN swaps.-Neither the base schedule nor any single NN swap schedule can be kept, but all departures can be made on time if a pair of trips, j and $j + 1$, make a NN swap and another distinct pair of trips, k and $k + 1$, make a NN swap. The probability of this outcome, P_{NNijk} , can be shown to be

$$P_{NNijk} = \frac{P_{NNi} P_{NNk}}{R_u} \quad (9)$$

The probability of all events of this type is R_{2NN} , given by

$$R_{2NN} = \sum_{j=1}^{m-3} \sum_{k=j+2}^{m-1} P_{NNijk} \quad (10)$$

4. Three NN swaps.-The base schedule cannot be kept as is or with one or two NN swaps, but all departures can be made on time with three distinct NN swaps. If the trip pairs are trips i and $i + 1$, j and $j + 1$, and k and $k + 1$, the probability of this outcome, P_{NNijk} , can be shown to be

$$P_{NNijk} = \frac{P_{NNi} P_{NNj} P_{NNk}}{R_u} \quad (11)$$

and the total probability of outcomes of this type, R_{3NN} , is

$$R_{3NN} = \sum_{i=1}^{m-5} \sum_{j=i+2}^{m-3} \sum_{k=j+2}^{m-1} P_{NNijk} \quad (12)$$

5. 2-3-1 swap.-If bus 1 arrives too late for both trips 1 and 2, but on time for trip 3, while bus 2 arrives early enough to make trip 1 and bus 3 arrives early enough for trip 2, a "2-3-1" swap (with bus 2 serving trip

1, bus 3 serving trip 2, and bus 1 serving trip 3) can occur, enabling all three trips to be made on time. Because bus 1 is too late for trips 1 and 2, neither the base schedule nor a schedule with only NN-type swaps can be kept. The outcome in which the base schedule cannot be kept as is or with NN swaps but can be kept with a single 2-3-1 swap involving trips k , $k + 1$, and $k + 2$ has the probability P_{231k} , which can be shown to be

$$P_{231k} = \frac{P(t_{k+1} \leq D_k)P(t_{k+2} \leq D_{k+1})P(D_{k+1} < t_k \leq D_{k+2})R_u}{r_k r_{k+1} r_{k+2}} \dots \dots \dots (13)$$

The total probability of outcomes of this type, R_{231} , is

$$R_{231} = \sum_{k=1}^{m-2} P_{231k} \quad \quad \quad (14)$$

6. 3-1-2 swap.-In this outcome, neither buses 1 nor 2 arrive early enough to serve trip 1, while bus 3 is early enough for trip 1, bus 1 is early enough for trip 2, and bus 2 is early enough for trip 3. The probability that the base schedule thus modified by a 3-1-2 swap involving trips k , $k + 1$, and $k + 2$ can be kept (and that none of the previously mentioned schedules can be kept) is

$$P_{312k} = \frac{P(t_{k+2} \leq D_k)P(D_k < t_{k+1} \leq D_{k+1})P(D_{k+1} < t_k \leq D_{k+2})R_u}{r_k r_{k+1} r_{k+2}} \dots \dots \dots (15)$$

The total probability of outcomes of this type, R_{312} , is

$$R_{312} = \sum_{k=1}^{m-2} P_{312k} \quad \quad \quad (16)$$

7. 3-2-1 swap.-This outcome is like a 3-1-2 swap, except that bus 1 arrives too late for trip 2 but early enough for trip 3, and bus 2 arrives early enough for trip 2, with bus 3 still early enough to make trip 1. P_{321k} , the probability of a single outcome in this category involving trips k , $k + 1$, and $k + 2$, is

$$P_{321k} = \frac{P(t_{k+2} \leq D_k)P(D_k < t_{k+1} \leq D_{k+1})P(D_{k+1} < t_k \leq D_{k+2})R_u}{r_k r_{k+1} r_{k+2}} \quad \quad \quad (17)$$

The total probability of outcomes of this type, R_{321} , is

$$R_{321} = \sum_{k=1}^{m-2} P_{321k} \quad \quad \quad (18)$$

8. Other combinations of two distinct swaps.-The pair of NN swaps, presented earlier, is a combination of two distinct "basic" swaps that can make it possible to meet the trip schedule when a single basic swap is insufficient. There are 15 other pairs involving the four basic types of swaps examined earlier (NN, 2-3-1, 3-1-2, and 3-2-1 swaps). The probability of a particular outcome involving a distinct pair of swaps can be shown, as in the double NN swap case, to be

$$P_{11}P_{22} \quad \quad \quad (19)$$

in which P_{s1} and P_{s2} = probabilities of the outcomes involving the single swaps. The total contribution of the outcomes of these combinations to the probability that the trip schedule can be met is simply the sum of the probabilities of all the distinct outcomes.

While the outcomes included in the preceding list constitute only a small fraction of all possible outcomes, there is reason to believe that they represent the most likely outcomes. Outcomes not included have a very low probability of occurrence because they involve many swaps or because they involve a bus being too late for its own scheduled trip as well as for at least the next two trips.

A further enhancement to the reliability estimate was made by giving special treatment when there are simultaneous scheduled departures. Trips with the same departure time cannot cover for each other (if a bus is too late for one, it is too late for the others), and so all swaps involving simultaneous departures will have zero probability. Therefore, each group of simultaneous departures was considered as a "supertrip" for purposes of defining the various swaps. Any outcome containing a swap that includes a supertrip is then expanded to include the whole set of outcomes in which the supertrip is replaced by one of its constituent trips. The expanded outcomes that contain two swaps that both include the same supertrip can be included in the probability sum as long as the supertrip is not replaced in both swaps by the same constituent trip.

This lower bound approximation of the pooled reliability can be inexpensively computed, even for a moderately large system. The number of computations involved without the simultaneous departures modification has been approximated as $(5/6)m^3 + 42m^2 + 30m$. Thus, for a system with 50 departures, there are approximately 2×10^5 computations. The simultaneous departure modification adds somewhat to this computational burden. For especially large systems, it may be necessary to eliminate the triple NN swap (which accounts for the m^3 term). The application reported herein, while including triple NN swaps, omitted outcomes involving 3-2-1 swaps since they were estimated to be about an order of magnitude less likely than 3-1-2 swap outcomes and two orders of magnitude less likely than 2-3-1 swap outcomes.

APPLICATION

The Sullivan Square Station of the Massachusetts Bay Transportation Authority's Orange Line serves as the terminus for 9 bus routes, all of which can be categorized as feeders, although one also performs a cross-town function. One of these routes operates very infrequently, which made data gathering difficult. Therefore, this study focuses on the eight major routes terminating at that station during the 3-6 p.m. peak. Between them, these routes have 119 departures from Sullivan Square during that period.

Round trip run times were observed on four different weekdays, yielding between 16 and 44 observations per route (26 observations on average), from which run time histograms for each route were constructed with one minute intervals. The histogram of each route was used directly as the run time distribution for every trip made on that

TABLE 1-Unpooled versus Pooled Reliability for Different Numbers of Buses

Number of buses (1)	Unpooled reliability (2)	Pooled reliability (3)
51	1.0	1.0
50	0.9412	1.0
49	0.8082	1.0
48	0.6041	1.0
47	0.3773	1.0
46	0.1579	1.0
46 ^a	0.1323	
45	0.0554	1.0
44	0.0053	1.0
43	0.0001	0.9592
42	0.0000	0.6803
41	0.0000	0.1842
40	0.0000	0.0066

^aCurrent system; buses not optimally deployed.

route. The routes are relatively short, with mean round trip run times of 40-67 min, and relatively reliable, with standard deviations of run time between 2.5 and 5.1 min (3.4 min on average). The data do not include situations of vehicle breakdowns or other major crises, and so the results obtained are relevant to operation under normal conditions.

The current schedule uses 46 buses and involves no interlining except among two routes where every vehicle alternates cyclically between the two routes. The reliability of the current schedule, without pooling, was calculated to be 13.2%. (Recall that our definition of reliability, the probability that every trip can be made on time, is quite strict, so that although the preceding reliability is low, operations appear quite smooth with late departures occurring only occasionally.) Redeploying one bus from one route to another increased the unpooled reliability slightly to 15.8%. This latter figure is used as a basis of comparison for comparing pooled to unpooled performance.

Table 1 shows the unpooled reliability and lower bound estimates of the pooled reliability of the system for different numbers of buses. In computing both pooled and unpooled reliability it was assumed that the first trip made by each bus was made with 100% reliability. In computing unpooled reliability, buses were optimally deployed among routes, and interlining was kept restricted to the pair of routes on which it is now practiced.

The current operating plan (unpooled, limited interlining) has, as mentioned earlier, a 15.8% reliability with the currently used 46 buses. It would need 51 buses to have 100% reliability, and would have 0% reliability with 42 or fewer buses. Under pooled operation, however, only 44 buses, two fewer than the number now used for the 8 routes, are needed for the reliability to be 100%. With only 41 buses, the pooled reliability is at least 18.4%, showing a possible savings of 5 of 46 buses without worsening reliability. Comparing the number of buses needed to achieve 100% reliability, there is a savings of 7 of 51 buses.

TABLE 2.-Contribution to Pooled Reliability of Base Schedule and of Different Classes of Swaps

Type of schedule (1)	Number of Buses				
	41 (2)	42 (3)	43 (4)	44 (5)	45 (6)
Base	0.0412	0.3124	0.6666	0.8145	1.0000
Nearest neighbor swap	0.1056	0.2717	0.2321	0.1685	0.0000
Pairs of NN swaps	0.0049	0.0303	0.0200	0.0106	0.0000
Triplets of NN swaps	0.0000	0.0001	0.0002	0.0002	0.0000
2/3/1 swaps	0.0226	0.0442	0.0326	0.0000	0.0000
Pairs of 2/3/1 swaps	0.0002	0.0004	0.0000	0.0000	0.0000
3/1/2 swaps	0.0070	0.0097	0.0029	0.0000	0.0000
Pairs of 3/1/2 swaps	0.0000	0.0000	0.0000	0.0000	0.0000
NN swap & 2/3/1 swap	0.0020	0.0094	0.0044	0.0000	0.0000
NN swap & 3/1/2 swap	0.0006	0.0019	0.0004	0.0000	0.0000
2/3/1 swap & 3/1/2 swap	0.0001	0.0002	0.0000	0.0000	0.0000
Total pooled reliability	0.1842	0.6803	0.9592	1.0000	1.0000

Table 2 shows the contribution to the pooled reliability of the various outcomes considered in the lower bound estimate, i.e., the base schedule outcome and the various swap outcomes described earlier. The outcomes contributing the most were the base schedule, the single and double NN swap, and the 2-3-1 swap. Contributing the least were pairs of 3-1-2 swaps, which contributed nothing (to four decimal places).

From the results presented in Table 2, it is possible to isolate the interlining benefit from the swapping benefit. Recall that the "base schedule reliability" reported in Table 2 is the probability of keeping, without swapping, a base schedule that is constructed with no limits on interlining. The difference between unpooled reliability reported in Table 1 and the base schedule reliability reported in Table 2 can thus be interpreted as the interlining benefit of pooling. The swapping benefit (actually a lower bound estimate of the swapping benefit) is then the difference between the base schedule reliability and the total pooled reliability estimate. Fig. 1 summarizes the reliability benefit attributable to interlining and to swapping for different numbers of buses. For 51 or more buses, unpooled operation has 100% reliability and so neither interlining nor swapping contribute anything to reliability. As the number of buses decreases to 45, unpooled reliability drops to only 5.5% while the reliability contribution of interlining grows to 94.5%. Up to this point, swapping continues to contribute nothing to the overall reliability, since the base schedule reliability is still 100%. As the number of buses further

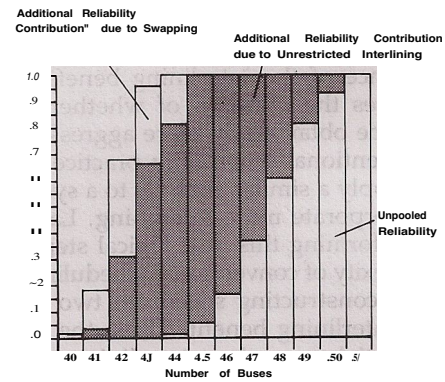


FIG. 1.-Reliability Benefit of Pooling: Interlining Benefit and Swapping Benefit

decreases, the unpooled reliability quickly becomes negligible, and the interlining contribution to the pooled reliability begins to fall while the reliability contribution of real-time swapping grows. With 42 buses, the interlining contribution to the pooled reliability is 31.2%, and the swapping contribution peaks at 36.8%. With fewer than 42 buses, both the interlining and the swapping contributions decrease, but the swapping contribution becomes more and more dominant.

In terms of buses saved, the interlining benefit accounts for the greater part of the pooling benefit. With 42 buses, a schedule with no interlining restrictions could maintain the existing reliability without any real-time swapping, compared to 41 with both interlining and swapping. Thus the interlining benefit accounts for 4 of the 5 buses that pooling can save at the current reliability level. For 100% reliability, only 45 buses are needed if unrestricted interlining but not swapping is allowed (compared to 44 with both interlining and swapping). In this case, 6 of the 7 buses saved through pooling can be attributed to interlining. These results indicate that, at least in the Sullivan Square case, the interlining benefit of pooling is its most important benefit.

CONCLUSIONS

From the results of this application, both the performance of the reliability estimation procedure and the practical value of the pooling strategy can be evaluated. The estimation procedure appears to be quite accurate, as even some of the classes of outcomes included in the reliability sum contributed nothing. Thus, it is likely that accounting for all the other possible ways of meeting the trip schedule would have added little to the reliability sum (however, we have not proven this). Modifying the procedure by specially treating simultaneous departures significantly enhanced its performance. The good performance of the procedure is also due in part to the small run time standard deviations for the routes

leaving Sullivan Square. With more variable run times, the more remote swapping possibilities take on greater probabilities, although we feel that for the full range of realistic values the estimate should still be good.

The magnitude of the pooling benefits, particularly since this system has quite reliable run times, speaks well for the value of the pooling strategy. The dominance of the interlining benefit over the swapping benefit, however, raises the question of whether the majority of the pooling benefit could be obtained by more aggressive use of interlining in the context of conventional scheduling practice. One way of testing this hypothesis is to apply a similar analysis to a system of routes whose present schedules incorporate more interlining. Lack of data prevented this research from performing this next logical step. However, it is our opinion that the complexity of conventional scheduling practice will hinder schedule makers from constructing schedules two to four times a year that achieve the full interlining benefit. Thus, the interlining benefit of pooling will be substantial even where interlining is already in extensive use. At the same time, pooling can greatly relieve the burden of schedulermakers who will only have to assign vehicles to a pool for a certain period of time to be sure that all trips served by that pool are covered rather than trying to fit every trip into the schedule of a particular vehicle. Furthermore, the swapping benefit should be more significant in systems with more run time variability.

While it has been the purpose of this paper to assess the benefits of pooling, mention should also be made of the difficulties an operator is likely to encounter in implementing this strategy. The main difficulty to be expected is in supervising the drivers. Only the dispatcher will know which driver went out on each trip (and he will only remember if he keeps a log), and so supervisors will have to check regularly with the dispatcher to know where each driver ought to be. Furthermore, with the conventional operating strategy, the driver has an incentive to return on time, since if he returns late he will lose some layover time. In pooled system, however, the consequences of a driver returning late are spread over the entire pool of drivers, and so the incentive to return on time is diminished, making supervision more important. Another difficulty is that the dispatcher will often have to adjust his simple first in/first out rule to account for drivers whose shift is about to end, in order to avoid having a driver out on a trip when his scheduled assignment ends, resulting in overtime costs.

Work rules should not present any more difficulty to the pooled system than to the unpooled system. "Run as directed" assignments exist now in the industry within the traditional work rules framework. However, pooling does not eliminate any work rule related scheduling problems, since the demand for drivers will still vary throughout the day, and assignments must still conform to work rules regarding length, spread, etc.

APPENDIX I-REFERENCES

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APPENDIX 11-NOTATION

The following symbols are used in this paper:

- D_i = scheduled departure time of trip i ;
 k = normalized schedule slack;
 m = number of trips;
 P = probability of specific outcome;
 R = reliability or (with subscript) reliability contribution;
 R_u = unpooled reliability;
 r_j = reliability of trip i ;
 s - schedule slack;
 t_j = end time of trip i 's predecessor;
 u_j = number of potential predecessors of trip i ;
 Φ = cumulative standard normal distribution; and
 σ = standard deviation of running time.